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THE UNIQUENESS RANGE OF 3-D VORTICITY HOT-WIRE PROBES: THEORETICAL LIMITS

A b s t r a c t

The theoretical borders of the uniqueness domains of the contemporary hot-wire probes specified for simultaneous 3-D vorticity measurement are analyzed, assuming the ideal sensors of infinite lengths. It is shown that 9-wire probes possessing 3 asymmetrical "T" arrays, theoretically reaches the maximal possible uniqueness cone of 26.5^0 half-angle at the sensors geometrical angles of 45^0. It is proven analytically that triple-orthogonal array possesses the largest uniqueness range among the triple- wire configurations, of 35.26^0 half-angle. However, adding the 4^{th} sensor, to each of the 3 "T" arrays of the 9-wire probe, what gives the 12-sensor probe, enlarges the uniqueness cone to the 38.8^0 half-angle, for the infinite wires mounted at geometrical angles of 45^0. In contrast to the triple-sensor probes and vorticity configurations possessing such arrays, the uniqueness range of quadruple probes and adequate vorticity probes can be enlarged by decreasing the sensor

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geometrical angles. For the angle of 40^0, the uniqueness cone half-angle reaches 41.8^0, for ideal sensor. Optimizing the interpretation procedure slightly encrease the half angle of the uniqueness cone, for only 0.4^0.

OBLAST JEDINSTVENOSTI TRODIMENZIONALNIH SONDI ZA MJERENJE VRTLOŽNOSTI - TEORIJSKE GRANICE

Izvod

Data je analiza teorijskih granica oblasti jedinstvenosti savremenih sondi za mjerenje tri komponente vrtložnosti, sa idealnim senzorom beskonačne dužine. Dokazano je da sonda sa 9 senzora odnosno tri T konfiguracije ima maksimalni konus jedinstvenosti 26.5^0, sa uglom senzora od 45^0. Trostruko ortogonalna konfiguracija ima najveći konus jedinstvenosti od 35.26^0 u poređenju sa ostalim konfiguracijama sa tri senzora. Međutim, dodavanjem četvrtog senzora na T konfiguracije sonde sa devet senzora, oblast jedinstvenosti se povećava na 38.8^0, sa uglom senzora od 45^0. Za razliku od sondi sa tri i devet senzora, oblast jedinstvenosti sondi sa četiri i dvanaest senzora se može povećati smanjenjem ugla senzora.

1. INTRODUCTION

A common approach in modeling the mechanism of hot-wire sensor cooling is based on the relation $U_e = f(E)$, between the "*effective cooling velocity*" U_e of the sensor by the surrounding flow, and the resulting output anemometer voltage drop E. Up to date, a variety of less or more accurate expressions for both the U_e and $f(E)$ have been formulated, as can be seen in **Bruun 1995, Vukoslavčević and Petrović 2000**, etc. However, the fourth-order polynomial

$$f(E) = a_0 + a_1 \cdot E + a_2 \cdot E^2 + a_3 \cdot E^3 + a_4 \cdot E^4. \qquad (1.1)$$

is generally accepted as the most accurate representation of $f(E)$. The effective cooling velocity is a complex function of the fluid velocity vector and calibration constants. The most frequently used are the components U, V and W, defined in the coordinate system fixed to the probe stem, (Fig. 1a,c) and the fluid velocity vector components that correspond to the local coordinate system of the sensor itself: normal component U_n orthogonal to the wire and lying in the prongs plane, binormal component U_b orthogonal to the plane formed by the wire and its prongs, and the tangential component U_t collinear to the wire axis, Fig. 1b, i.e.

$$U_e = f_{fix}(U, V, W) = f_{loc}(U_n, U_b, U_t). \tag{1.2}$$

Obviously, components U, V, W and U_n, U_b, U_t are related, because they represent the same fluid velocity vector in different coordinate systems $Oxyz$ and $Onbt$, i.e.

$$\vec{U}_0 = U \cdot \vec{i} + V \cdot \vec{j} + W \cdot \vec{k} = U_n \cdot \vec{n}_0 + U_b \cdot \vec{b}_0 + U_t \cdot \vec{t}_0. \tag{1.3}$$

The simplest example corresponds to the so-called "*normal single-wire probe*", with a single sensor orthogonal according to the prongs direction in horizontal plane. For such probe, both of these two coordinate systems are identical and, therefore, $U_n = U, U_b = V$ and $U_t = W$, Fig. 1a. However, for the wire inclined at geometrical angle α toward to the normal to the prongs direction, placed in an arbitrary plane forming an angle β toward the horizontal plane, these relations are functions of the parameters defining the wire orientation (see figs. 1b, c).

The basic theoretical concept of hot-wire probe response assumes the infinite length of the probe sensors. Consequently, conductive heat transfer from hot-wire ends to the prongs can be neglected, as well as the prongs aerodynamic blockage that may result in the local acceleration of the real flow in the sensor's vicinity (i.e. within the probe sensing volume). It means that, for an infinitely long (ideal) wire, the effective cooling velocity U_e is equal to the component U_N of the fluid velocity vector, orthogonal to the sensor's longitudinal axis (see figs. 1a,b),

$$U_N^2 = U_n^2 + U_b^2. \tag{1.4}$$

Calculation of U_N and, therefore, U_e also, is very simple in this case: the intensity U_0 of the instantaneous fluid velocity vector $\vec{U}_0$ multiplied

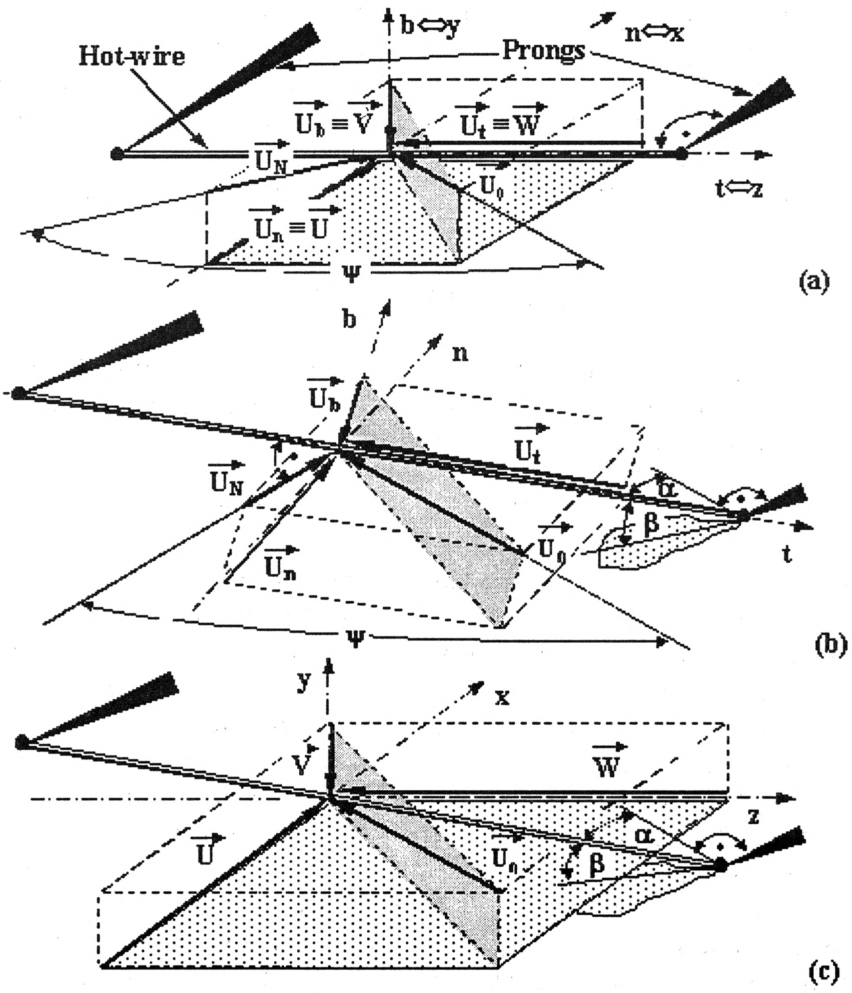
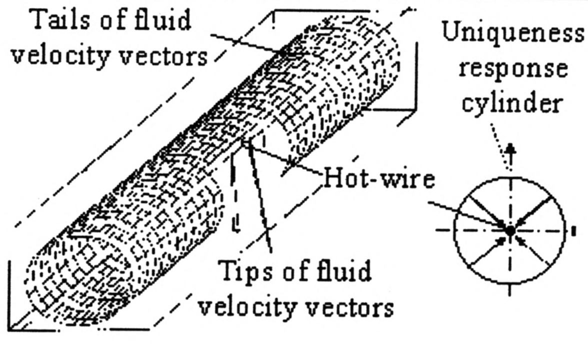


Fig. 1 A fluid velocity vector $\vec{U_0}$ represented in different Cartesian coordinate systems, assuming the probes aligned to the mean flow direction defined by x-axis: (a) a normal single-wire probe lying in the horizontal plane,β giving $U = U_n, V = U_b$ and $U_t = W$; (b) a local system fixed to a wire inclined at geometrical angle α and placed in a plane forming an angle β toward the horizontal plane and (c) a coordinate system fixed to the mean flow direction.



(a) (b)

Fig. 2 Fluid velocity vectors giving the same response of an infinite hot-wire, according to **Willmarth 1985**: (a) the axonometric view (b) the side view. Adapted from: **Dobbeling, Lenze and Leuckel 1990**.

by the *cosine* of angle ψ (see figs. 1a,b) formed by the vector $\vec{U}_0$ and the normal to the wire axis

$$U_e^2 = U_n^2 + U_b^2 = U_N^2 = (U_0 \cdot cos\psi)^2. \tag{1.5}$$

Because of the *cosine* function included, expression (1.5) is usually designated as "*the cosine law*". It shows that an infinitely large number of different U_n and $U_b(U_0$ and $\psi)$ combinations exist, which give the same U_e. Directly, different fluid velocity vectors may generate the same hot-wire response. Therefore, each specific set of hot-wire anemometer output signals can give multiple solutions, i.e. "*measured*" velocity components - the well-known "*uniqueness*" or "*rectification*" problem (**Tutu and Chevray 1975**). Its complexity grows up with increasing the number of sensors, making the analysis of multiple hot-wire probes directional response, which is in the focus of the present paper, to be a common problem related to experimental researching of turbulence.

An illustrative graphical representation of an ideal infinitely long hot-wire response to the possible fluid velocity vector directions, which follows the "*cosine law*" (1.5), is given by **Willmarth 1985**, Fig. 2.

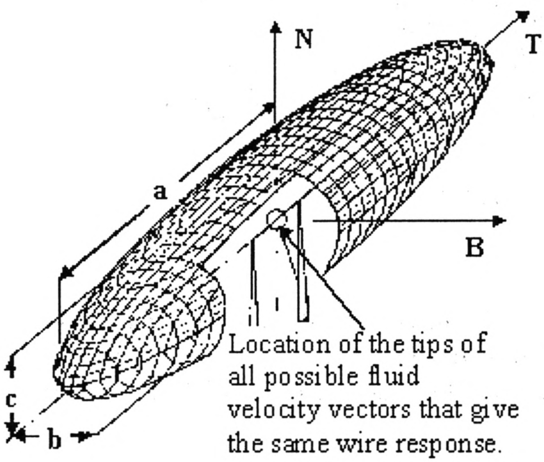


Fig. 3 Fluid velocities giving the same signal of a finite-length (real) wire. Source: **Dobbeling, Lenze and Leuckel 1990a.**

He showed that infinite number of possible fluid velocity vectors of various magnitudes and directions give the same wire response. Their tips lay in the center of the sensor and the tails are on the cylinder of infinite length (Fig. 2a). In the case of two sensors, an infinite number of possible fluid velocity vectors, giving the same wire response, will be defined by the intersection curve of those two cylinders. Finally, in the case of three sensor probe, there could be up to 8 intersection points.

In comparison to the ideal hot-wire, the cooling mechanism of a real finite-length sensor is very complex and has to be described by a formula that accounts for all 3 fluid velocity vector components. The empirical law of **Jorgensen 1971**

$$U_e^2 = U_n^2 + k^2 \cdot U_t^2 + h^2 \cdot U_b^2. \tag{1.6}$$

still belongs to the topics in highly accurate description of hot-wire sensor cooling mechanism. Graphical illustration of (1.6) is an offline contraction of a rotational ellipsoid, Fig. 3, with half-axes a, b, c giving $k = c/b$ and $h = c/a$ for the calibration constants in (1.6).

Although the response of a finite-length sensor is much more complex in comparison to the ideal wire, general conclusions relating the probe uniqueness range are similar. As it can be seen from (1.6) and Fig. 3, an infinite number of fluid velocity vectors of different intensities and directions also exist for a real sensor, giving the same U_e. And, in full analogue to the ideal wire, an infinite number of possible fluid velocity vectors with the same intensity, but of the various directions giving the same wire response, exist in the plane normal to the wire axis. It follows that basic analysis, providing general relationships between the 3-D probe uniqueness range and its geometry, can be successfully performed assuming infinite ideal sensors.

2. PRESENT SITUATION

To measure simultaneously the 3-D turbulent vorticity vectors, very accurate measurements of all 3 fluctuating components of the instantaneous fluid velocity vectors are needed, demanding at least the same number of hot-wires. However, as it is explained in the previous chapter, hot-wire's sensitivity toward the fluid velocity direction is not unique, what limits the applicability of each vorticity-type and velocity-type 3-D probe to a certain angular range, where their output signals enable unique determination of the fluid velocity vectors.

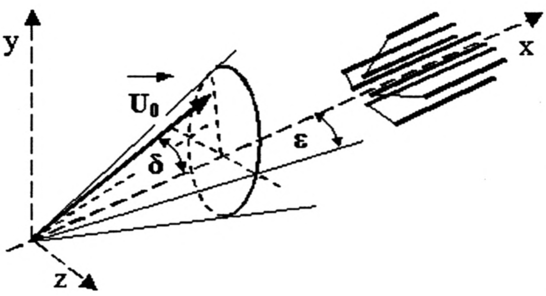
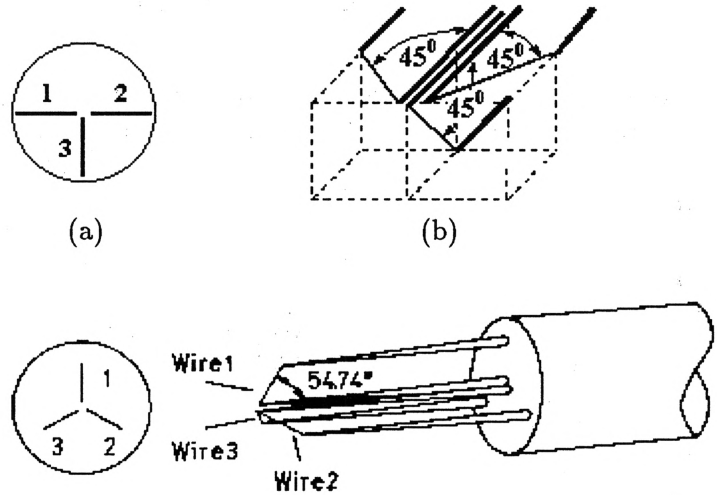


Fig. 4 A sketch defining the uniqueness cone.

Geometrical shape of such angular range depends primarily on hot-wires configuration and the applied signal interpretation procedure, but also on the manufacturing technology, magnitudes of the measured flow velocity vectors, etc.

However, this "*uniqueness domain*" (also known as the "*uniqueness range*") is ordinarily described by a conical surface known as the "*uniqueness cone*", Fig. 4, which half-angle ε is a direct measure of a probe angular applicability. Theoretically, each specified probe can be applied only in the flow which turbulence level is small enough to guarantee that maximal angle δ of the instantaneous fluid velocity vectors toward the probe axis is smaller than ε. In practice, the problem is resolved by introducing the so-called "*threshold*" involved in the signal interpretation procedure. This routine checks the instantaneous orientation of a measured fluid velocity vector, and rejects those falling out from the uniqueness cone, i.e. when $\delta > \varepsilon$ (for angles definitions see Fig. 4).



(a) (b)

(c) (d)

Fig. 5 Typical "*triple*" hot-wire configurations: (a) a front view of "*T*" probe; (b) an axonometric view of "*T*" configuration; (c) a front view of an orthogonal "*Mercedes*" - "*M*" geometry and (d) an axonometric view of "*M*" probe. Figs. (b) and (c) are adapted from **Rosemann 1989**.

It is known that *Jorgensen's* empirical formula, as well as the other most common hot-wire cooling equations, may not be very accurate representations of hot-wire probe response to the fluid velocity close to the the angular uniqueness range borders. Therefore, especially when an extremely accurate measurements of fluid velocity are needed, as for vorticity mesurements, the "*acceptance cone*" half-angle may be intentionally decreased in comparison to the analogue "*uniqueness cone*" half-angle. Unfortunately, involving the threshold in the interpretation procedure of 3-D hot-wire probe output anemometer signals can not completely resolve the problem. Rejecting the data lying out from the uniqueness cone distorts the required information related to the flow of interest, especially the higher-order statistics. Therefore, the additional efforts have been made to enlarge the uniqueness domain of 3-D triple-wire probes.

Up-to-date, two main classic types of triple-wire probes, capable of simultaneous 3-D fluid velocity vectors measurement, have been designed: "*T*" configuration (Fig. 5a,b) and "*orthogonal triple-sensor probe*" (Fig. 5c,d), designated also as "*Mercedes*" or "*M*" type in the further text. Standard "*T*"-configured hot-wires are mounted at 45^0 toward the probe longitudinal axis (Fig. 5b). Two sensors lay in the same plane forming "*V*" geometry, while the 3^{-rd} is in the vertical plane. **Vukoslavčević and Wallace 1983** showed that half-angle of uniqueness cone of "*T*" probe is 26.5^0 for the ideal probe possessing sensors of infinite length, and only 17.5^0 for their real miniature probe at velocity magnitude below 1 m/s. In spite of this problem, "*T*" configuration is still used as an array of the 9-wire vorticity probe, because of the simple design and signal interpretation procedure.

The "Mercedes" geometry (Fig. 5c,d) is the most commonly used triple-sensor probe. It contains 3 mutually orthogonal sensors lying in the planes that form 3 identical angles of 120^0. Simple geometrical analysis shows that sensors are inclined at $\alpha = 54.74^0$ toward the probe longitudinal axis, giving the geometrical wires angle of 35.26^0.

An illustrative mathematical explanation of the uniqueness problem is given by **Lekakis, Adrian and Jones 1989**: The non-uniqueness of the solution of hot-wires response equations can not be totally eliminated because the **Jorgensen's 1971** equations are invariant under changes of the sign of velocity vector. Hence, the ambiguity is at least two-fold, for any sensor arrangement and for any number of cylindrical sensors". Also, if one wish to measure the turbulence fluctuations, hot-wire sensors has to be out of the prongs wakes, as well as out of the wakes of other sensors. Therefore, with classic hot-wire technology, whatever the large number of sensors is used, the angular applicability of a stationary probe is limited to a hemisphere. Up-to-date, only **Holzapfel, Lenze and Leuckel 1994** have reported the quintuple (5-wire) probe, which uniqueness domain maybe can reach this limit. Their belief is based on the fact that at least signals of the 3 sensors (from the total number of 5 wires placed in the planes that mutually forms angles of 72^0) are out from the wakes of the other hot-wires and prongs within a hemisphere. There were some attempts in the past, by **Acrivellis 1980** and **Kawal, Shokr and Keffer 1983** for example, related to the application of the special non-orthogonal triple-wire configurations for studding the flows whose turbulence levels are higher than those in which the standard triple-orthogonal (*Mercedes*) probes can be successfully applied. Analogue approach, for the crossed hot-wire probes containing two sensors, was reported by **Blanco-Marigota, Ballesteros-Tajadura and Santolaria 1998**.

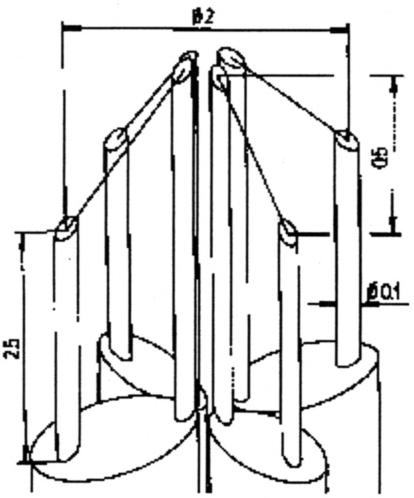


Fig. 6 "Plus" probe of **Dobbeling, Lenze and Leuckel 1990a**, similar to the arrays of the 12-wire vorticity probes.

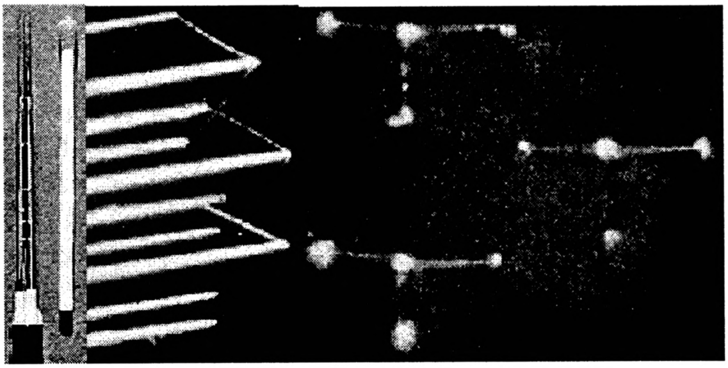
However, **Lekakis, Adrian and Jones 1989** and **Roseman, Stager and Kreplin 1996** reported that the uniqueness range of the triple hot-wire probes can not be extended beyond the uniqueness domain border of the triple-orthogonal (*Mercedes*) configuration, theoretically found to be 35.26^0. Also, they have found that angular sensitivity of the triple probes decreases with increasing the wires angle toward the probes axis, what means that sharper probe provides the smaller uniqueness range and higher angular sensitivity toward fluid velocity and vice versa.

Therefore, at present level of standard hot-wires technology development, it has become generally accepted that further enlargement of the uniqueness domain of 3-D hot-wire probes demands introduction of additional wires. "*Quadrate*" configurations with 4 sensors, initially developed for the longitudinal vorticity measurement by **Kovasznay 1950** and finalized by **Vukoslavčević and Wallace 1981**, also enable simultaneous 3-D turbulent velocity vector measurements. However, the most common contemporary quadruple configuration, specified for 3-D velocity measurement, is the so-called "*plus*" probe sketched in Fig. 6. It is also applied as an array of the most sophisticated vorticity probes possessing 12-sensors.

3. THE OBJECTIVE

The ideal sensors, of indefinite lengths, and therefore without the prongs influence, provide the largest possible acceptability range for the multiple probe of each specific geometry. Thus, such kind of analysis should give the upper limits, i.e. the maximal possible boundaries of angular applicability of each specific hot-wire probe configuration.

Obviously, these boundaries can't be reached by a real probe possessing finite hot-wires. Still, the goal of the probe designers should be to provide the uniqueness cone that is close as possible to these theoretical borders. This can be achieved by the optimal probe design, combined by the sophisticated signal interpretation procedure.



(a) (b) (c)

Fig.7 The 9-wire probe: (a) a side-view, (b) probe assembly and (c) a front view. Adapted from: **Vukoslavčević, Wallace and Balint 1990**.

Present analysis is focused to the upper limits of the uniqueness ranges of 2 probes specified for simultaneous 3-D measurement of the turbulent velocity, velocity gradient and vorticity vectors: the 9-sensor probe of **Vukoslavčević, Wallace and Balint 1991** presented in Fig. 7, and the 12-wire probe of **Vukoslavčević and Wallace 1996**, which photograph is presented in Fig. 8.

Following the standard measuring principle of most contemporary vorticity hot-wire probes, which assumes the simultaneous 3-D fluid velocity measurement at 3 adjacent locations lying in the plane orthogonal to the main flow direction, both the 9-wire and the 12-sensor probes consist of the 3 mutually identical arrays. As it can be seen in Figs. 7 and 8, the numbers and configurations of hot-wire sensors included in each array of these vorticity probes are different: 3 wires of "T"-configuration and 4 sensors of "+" geometry, respectively.

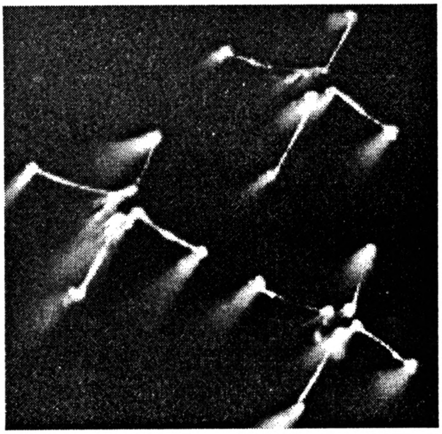
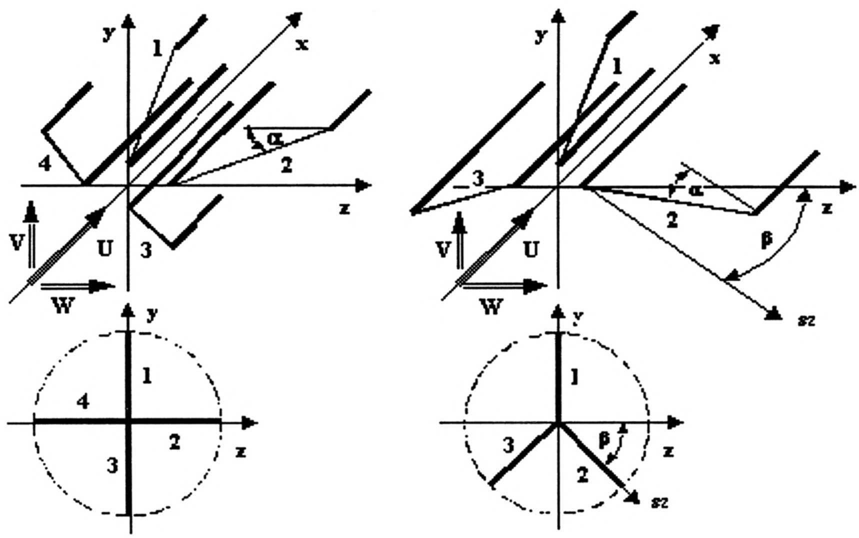


Fig.8 The 12-sensor probe of **Vukoslavčević and Wallace 1996**.

Each single array of these vorticity configurations also represents an independent probe for 3- D measurement of the fluctuating fluid velocity vectors, which uniqueness cone defines the applicability of the complete vorticity probe. Therefore, our analysis is concentrated to the probes specified to measure 3-D velocity vectors, primarily "T" and "+" configurations, but also to the triple-orthogonal geometry array, included in the 9-wire vorticity probe of **Honkan 1993**.

A special algorithm was developed following the 9-wire and 12-sensor probes interpretation procedure of **Vukoslavčević and Wallace 1996** and its optimized variant, proposed by **Petrović 1996**.

4. PHYSICAL BACKGROUND



(a) (b)

Fig. 9 A sketch of the coordinate system for: (a) "T" and "+" probe and (b) for Mercedes geometry.

The cooling mechanism of infinite hot-wires based on the "*cosine law*" (1.5), gives the following formulas for the sensors of the "T" and "+" geometry and coordinate system drawn in Fig. 9a:

$$U_{e1}^2 = U_{N1}^2 = (U \cdot cos\alpha - V \cdot sin\alpha)^2 + W^2, \tag{4.1}$$

$$U_{e2}^2 = U_{N2}^2 = (U \cdot cos\alpha - W \cdot sin\alpha)^2 + V^2, \tag{4.2}$$

$$U_{e3}^2 = U_{N3}^2 = (U \cdot cos\alpha + V \cdot sin\alpha)^2 + W^2, \tag{4.3}$$

$$U_{e4}^2 = U_{N4}^2 = (U \cdot cos\alpha + W \cdot sin\alpha)^2 + V^2. \tag{4.4}$$

By analogue, adequate expressions were developed for the triple "*Me-*

rcedes" probe, Fig. 9b:

$$U_{e1}^2 = U_{N1}^2 = (U \cdot cos\alpha - V \cdot sin\alpha)^2 + W^2, \tag{4.5}$$

$$U_{e2}^2 = U_{N2}^2 = (Ucos\alpha + Vsin\alpha sin\beta - Wsin\alpha cos\beta)^2 + (Vcos\beta + Wsin\beta)^2, \tag{4.6}$$

$$U_{e3}^2 = U_{N3}^2 = (Ucos\alpha + Vsin\alpha sin\beta + Wsin\alpha cos\beta)^2 + (Vcos\beta - Wsin\beta)^2. \tag{4.7}$$

It is suitable to define the uniqueness domain border of a given hot-wire probe in the polar coordinate system, over the full range of $0 - 360^0$ of a polar angle θ that is related to the ratio of the spanwise fluid velocity components

$$\tan\theta = W/V. \tag{4.8}$$

Keeping the θ (i.e. the ratio W/V) constant, and increasing jaw angle φ of the fluid velocity vector, defined as

$$tg\varphi = V_s/U = \sqrt{V^2 + W^2}/U, \tag{4.9}$$

the maximal angle φ_{CR}, which still guarantee the unique solution of hot-wire response equations can be found for each given angle θ. This φ_{CR} represents the border angle of the probe angular uniqueness domain, for the chosen θ. For the geometrically asymmetric probes, the angular uniqueness domain border, defined by φ_{CR}, is usually different in $x - O - z$ plane ($V = 0 \Leftrightarrow \theta = 90^0, 270^0$) and $x - O - y$ plane ($W = 0 \Leftrightarrow \theta = 0^0, 180^0$), and depends on the ratio of W/V or angle θ, according to expression (4.8). This way, a uniqueness domain, either symmetric or asymmetric, depending primarily on the possible probe geometrical asymmetry, can be defined for each specific hot-wire probe geometry.

However, it is more convenient in the experimental practice to use the so-called "*uniqueness cone*", defined by the half-angle ε, sketched in Fig. 4, which is defined as the minimal value of φ_{CR} in the whole range of $\theta(0 - 360^0)$. Therefore, the uniqueness cone is the conical surface of the largest possible half-angle ε, that can be drawn within the more complex surface representing the angular uniqueness domain of hot-wire probe.

5. ANALYTICAL SOLUTIONS FOR THE TRIPLE-WIRE CONFIGURATIONS

5.1 Analytical solutions for the "T" probe

Rejecting one of the two available wires (no. 1 or no. 3) placed in the vertical plane of the "+" probe, sketched in Fig. 9a, two triple-wire "T" configurations arise, which responses are described by the expressions (4.1), (4.2) and (4.4), or the formulas (4.2), (4.3) and (4.4). The analytical solutions presented in further text are formulated following **Vukoslavčević and Wallace 1983**, who showed that critical case for "T" probe, possessing the lower sensor no. 3 in the vertical plane, arises at $W = 0$ and high positive V component.

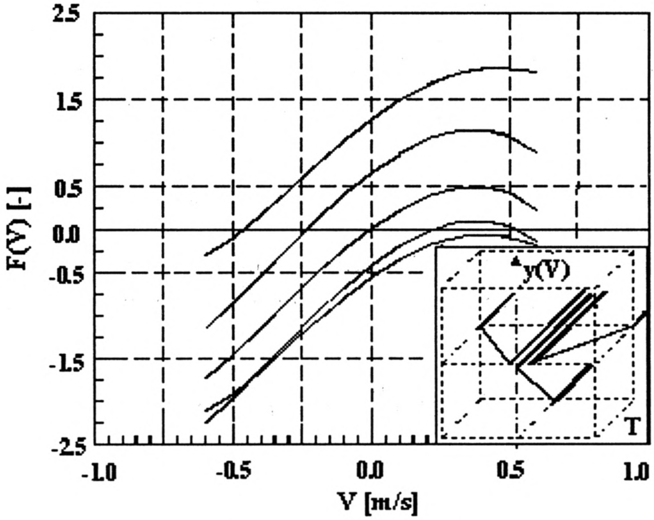


Fig. 10: Functions $F = F(V)$, for various V/U ratios. Source: **Vukoslavčević and Wallace 1996.**

1^0 The first special case, $W = 0 (\theta = 0^0, 180^0)$

A "T" probe possessing wires no. 1, 2 and 4 (Fig. 9a) described by the equations (4.1), (4.2) and (4.4), gives for $W = 0$:

$$U_{e1}^2 = (U cos\alpha - V sin\alpha)^2 = U_2 \cdot (cos^2\alpha - k_V sin2\alpha + k_V^2 \cdot sin^2\alpha), \quad (5.1)$$

$$U_{e2}^2 = U_{e4}^2 = (U \cdot cos\alpha)^2 + V^2 = U^2 \cdot (cos^2\alpha + k_V^2), \quad (5.2)$$

where $k_V = V/U = \tan\varphi$. Assuming $U = 1$, the expressions (5.1) and (5.2) give the following equation:

$$(U_{e1}^2 - U_{e2}^2 sin^2\alpha)\cdot k_V^2 + U_{e2}^2 sin(2\alpha)\cdot k_V + (U_{e1}^2 - U_{e2}^2)\cdot cos^2\alpha = 0 \quad (5.3)$$

Under assumption of $U = 1$, the function $f(k_V)$ (5.3) becomes $f(V)$. It is of the 2^{-nd} order for k_V, what results in 2 possible (real or imaginary) roots for k_V or V component. When the discriminant of (5.3)

$$D = (U_{e2}^2 sin2\alpha)^2 - 4(U_{e1}^2 - U_{e2}^2 sin^2\alpha)\cdot k_V + (U_{e1}^2 - U_{e2}^2)\cdot cos^2\alpha \quad (5.4)$$

equals to zero, the characteristic function (5.3) touches the abscissa, as shown in the Fig. 10, giving only one real root, $k_V = k_{Vcr}$. The uniqueness range is, therefore, defined by $k_V < k_{Vcr}$, or $V < V_cr$. There will be only one intersection of characteristic function $f(V)$ inside that range, i.e. the only one real root. Above this range 2 real roots can appear. The expression (5.4) can be reduced, after simple transformation, to

$$D = U_{e1}^2[U_{e1}^2 - U_{e2}^2(1 + sin^2\alpha)] \quad (5.5)$$

After introducing the definitions of U_{e1} and U_{e2}, according to (5.1) and (5.2), it is clear that borders of the uniqueness domain in this case are reached when

$$U_{e1}^2 = 0 \xrightarrow{(5.1)} k_{Vcr1} = ctg\alpha \quad (5.6)$$

and

$$U_{e1}^2 - U_{e2}^2\cdot(1 + sin^2\alpha) = 0 \xrightarrow{(5.1),(5.2)} k_{Vcr2} = -sin(2\alpha)/2 \quad (5.7)$$

for positive and negative V, respectively. The critical angle, defining the maximal spanwise velocity component with only one real root will be

$$\tan\varphi_{cr1,2} = \frac{\sqrt{V_{cr}^2 + W_{cr}^2}}{U} = \frac{\sqrt{V_{cr}^2}}{U} = k_{crV_{1,2}}, \quad (5.8)$$

giving a value of $\varphi_{cr1} = atan(cotan\alpha)$, for positive V and $\varphi_{cr2} = atan([-sin(2\alpha)/2]$, for negative V. For $\alpha = 45^0$, we have $\varphi_{cr1} = 45^0$

and $\varphi_{cr2} = -26.5^0$. The latter value agrees with **Vukoslavčević and Wallace 1983**, with the logical difference in the algebraic sign. They have reported $+26.5^0$, because their "T" probe, possessing the lower wire (i.e. sensors no. 2, 4 and 3 in Fig. 9a) was symmetrically oriented in comparison to our analyzed case, which assumes the "T" configuration with the upper sensor in the vertical plane, containing hot-wires no. 1, 2 and 4 according to Fig. 9a.

2^0 The second special case, $V = \pm W(\theta = 45^0, 135^0, 225^0, 315^0)$

In order to provide a referent "check points" for the results of our numerically simulated "T" hot-wire responses of the "T" probe, according the different fluid velocity vectors, we have extended the analysis to additional cases. For the fluid velocity vector orientations giving $V = W$, the response equations (4.1), (4.2) and (4.4) of the "T" geometry transform to the two second-order equations:

$$U_{e1}^2 = U_{e2}^2 = (U \cdot cos\alpha - V \cdot sin\alpha)^2 + V^2 =$$

$$= U^2 \cdot (cos^2\alpha - k_V \cdot sin2\alpha + k_V^2 \cdot sin^2\alpha + k_V^2), \qquad (5.9)$$

$$U_{e4}^2 = (U \cdot cos\alpha + V \cdot sin\alpha)^2 + V^2 =$$

$$= U^2 \cdot (cos^2\alpha + k_V \cdot sin2\alpha + k_V^2 \cdot sin^2\alpha + k_V^2). \qquad (5.10)$$

These two equations give, after simple algebraic transformations, the following equation:

$$(sin^2\alpha + 1) \cdot (U_{e1}^2 - U_{e4}^2) \cdot k_V^2 + sin2\alpha \cdot (U_{e1}^2 + U_{e4}^2) \cdot k_V +$$

$$+ (U_{e1}^2 - U_{e4}^2) \cdot cos^2\alpha = 0. \qquad (5.11)$$

Thus, the problem is now focused to the solution of equation:

$$D = sin^2 2\alpha \cdot (U_{e1}^2 + U_{e4}^2) - 4 \cdot (sin^2\alpha + 1) \cdot cos^2\alpha \cdot (U_{e1}^2 - U_{e4}^2)^2 = 0 \quad (5.12)$$

which can be transformed, after introducing (5.9) and (5.10), to the fourth-order equation

$$D = [(sin^2\alpha + 1) \cdot k_V^2 - cos^2\alpha]^2 = 0. \qquad (5.13)$$

It follows that the borders of the uniqueness domain in this case are reached at:

$$k_{Vcr1,2} = \pm \frac{cos\alpha}{\sqrt{sin^2\alpha + 1}}. \qquad (5.14)$$

However, $V = W$, thus giving

$$k_{Vcr1,2} = \left(\frac{V}{U}\right)_{cr1,2} = \left(\frac{W}{U}\right)_{cr1,2}. \tag{5.15}$$

Therefore, the critical angle of the fluid velocity vector toward the probe axis is defined as

$$\tan\varphi_{cr} = \frac{\sqrt{V_{cr}^2 + W_{cr}^2}}{U} = \frac{\sqrt{2 \cdot V_{cr}^2}}{U} = \sqrt{2} \cdot k_{Vcr1,2} = \pm\frac{\sqrt{2} \cdot cos\alpha}{\sqrt{sin^2\alpha + 1}} \Rightarrow$$

$$\Rightarrow \varphi_{cr_{1,2}} = \pm atan\left(\frac{\sqrt{2} \cdot cos\alpha}{\sqrt{sin^2\alpha + 1}}\right). \tag{5.16}$$

The same φ_{cr} is obtained for $V = -W$. For the wire geometrical angle of interest, $\alpha = 45^0, \varphi_{cr} = \pm 39.2^0$.

3^0 The third special case, $V = 0(\theta = 90^0, 270^0)$

In this special case, a fluid velocity vector lies in the Oxz plane of the probe coordinate system sketched in Fig. 9a. For the fluid velocity vector orientations, satisfying the $V = 0$, the response equations (4.1), (4.2) and (4.4) of the "T" hot-wire configuration give the following formulas:

$$U_{e1}^2 = (U \cdot cos\alpha)^2 + W^2, \tag{5.17}$$

$$U_{e2}^2 = (U \cdot cos\alpha - W \cdot sin\alpha)^2, \tag{5.18}$$

$$U_{e4}^2 = (U \cdot cos\alpha + W \cdot sin\alpha)^2. \tag{5.19}$$

Now, in contrast to the previous 2 special cases, 3 hot-wire response equations exist for evaluation of 2 unknown velocity components U and $W(V = 0)$. Thus, it is possible to form 3 combinations consisting of 2 equations, in order to calculate U and W. Combining the formula (5.17) for the vertical wire no. 1 with (5.18) or (5.19) (sensors no. 2 or no. 4, Fig. 9a) gives the two sets of equations analogue to (5.1) and (5.2)) discussed in the case no. 1^0. Critical values for these combinations are therefore identical as in the case no 1^0. However, now we can use the outer (large) boundaries combining these 2 combinations (wires no. 1,

4 and 1, 2, respectively Fig. 9a). They give
$k_{Vcr1,2} = \pm ctan\alpha \Rightarrow \varphi_{cr1,2} = \pm(90^0 - \alpha)$, or

$$\varphi_{cr1,2} = \pm 45^0 \quad \text{for} \quad \alpha = 45^0. \tag{5.20}$$

If the expressions (5.18) and (5.19) for the horizontal sensors no. 2 and no. 4 (Fig. 9a) are combined, a typical "V" hot-wire configuration arise. However, whatever combination of the sensors is used, the same results for the critical $k_{Vcr}(\varphi_{cr1,2})$ values arise:

5.2 Analytical solutions for the "Mercedes" probe

Following **Rosemann 1989** and **Rosemann, Stager and Kreplin 1996**, among others, the most critical situations related to the angular uniqueness range of the "Mercedes" configuration (sketched in Fig. 9b) arise when the fluid velocity vector lies in the sensor's plane. Therefore, the problem can be analyzed in any of 3 different angular planes that mutually form the 120^0 angles. We have chosen the vertical plane containing the wire no. 1 (see Fig. 9b) defined by $W = 0$, for the analysis. In this case, expressions describing the hot-wire cooling velocities, i.e. (4.5), (4.6) and (4.7), reduces to:

$$U_{e1}^2 = (U \cdot cos\alpha - V \cdot sin\alpha)^2 = U^2(cos\alpha - k_V \cdot sin\alpha)^2 =$$
$$= U^2(cos^2\alpha - k_V \cdot sin2\alpha + k_V^2 \cdot sin\alpha), \tag{5.21}$$

and

$$U_{e2}^2 = U_{e3}^2 = \left(U \cdot cos\alpha + \frac{V}{2} \cdot sin\alpha\right)^2 + \frac{3}{4}V_2 =$$
$$= U^2 \cdot \left(cos^2\alpha + \frac{k_V}{2} \cdot sin2\alpha + \frac{k_V^2}{4} \cdot sin^2\alpha + \frac{3}{4} \cdot k_V^2\right). \tag{5.22}$$

Using the approach applied in the section 5.1, the following characteristic equation arises after combining (5.21) and (5.22):

$$\left(\frac{1}{4} \cdot sin^2\alpha \cdot U_{e1}^2 + \frac{3}{4} \cdot U_{e1}^2 - sin^2\alpha \cdot U_{e2}^2\right) \cdot k_V^2 +$$
$$+ sin2\alpha\left(\frac{U_{e1}^2}{2} + U_{e2}^2\right) \cdot k_V + cos^2\alpha(U_{e1}^2 - U_{e2}^2) = 0. \tag{5.23}$$

Again, the angular border of the uniqueness domain is reached when the discriminant of the characteristic equation ((5.23) in this case) equals zero:

$$D = U_{e1}^2 \cdot [U_{e1}^2 - (3 \cdot sin^2\alpha + 1) \cdot U_{e2}^2] = 0. \tag{5.24}$$

This condition is satisfied if one of these two factors is zero.

1^0 $U_{e1}^2 \overset{(5.21)}{\longrightarrow} cos\alpha - sin\alpha \cdot k_V = 0 \Rightarrow k_{Vcr1} = ctg\alpha \Rightarrow$

$$\Rightarrow \varphi_{cr1} = 90^0 - \alpha \tag{5.25}$$

2^0 $U_{e1}^2 - (3 \cdot sin^2\alpha + 1) \cdot U_{e2}^2 = 0$ (5.26)

After introducing definitions of the effective cooling velocities U_{e1} and U_{e2}, (5.21) and (5.22) respectively, the latter formula transforms in the more practical form

$$k_V \cdot (sin^2\alpha + 1) + sin2\alpha = 0 \Rightarrow k_{Vcr2} = -\frac{sin2\alpha}{1 + sin^2\alpha} \Rightarrow$$

$$\Rightarrow \varphi_{cr2} = atan\left(-\frac{sin2\alpha}{1 + sin^2\alpha}\right). \tag{5.27}$$

For the sensor geometrical angle $\alpha = 35.26^0$, which defines the orthogonal triple-wire (*Mercedes*) geometry, the uniqueness cone is limited by the smaller of these 2 values of $\varphi_{cr1,2}$(45^0 and -35.26^0, respectively), i.e. by $\varphi_{cr2} = -35.26^0$. Negative algebraic sign of φ_{cr2} corresponds to lower part of the *Mercedes* probe, where V component is negative. The same critical value, originating from a quite different approach, was also reported by **Dobbeling, Lenze and Leuckel 1990a.**

5.3 The optimal choice of hot-wires geometrical angles relating the uniqueness cone

Following **Vukoslavčević and Wallace 1983**, the optimal hot-wires geometrical angle α, which provides maximal half-angle ε of the "T" probe uniqueness cone, was found by equaling the first derivative of (5.7) to zero:

$$\frac{d(k_V^T)}{d\alpha} = 0 \Rightarrow cos(2\alpha) = 0 \Rightarrow \alpha_{OPT} = 45^0. \tag{5.28}$$

It follows that 9-wire probe, possessing 3 arrays of 3 "T" configured sensors mounted at 45^0 toward the probe axis provides the largest possible uniqueness cone for that configuration.

Similarly, the uniqueness cone borders of the "*Mercedes*" configuration were analyzed. Equaling the first derivative of the expression (5.27) gives the optimal geometrical angle of the sensors:

$$\frac{d(k_V^M)}{d\alpha} = \frac{1 - 3 \cdot sin^2\alpha}{(1 + sin^2\alpha)^2} = 0 \Rightarrow sin\alpha = \frac{\sqrt{3}}{3} \Rightarrow \alpha_{OPT} = 35.26^0. \quad (5.29)$$

For $\alpha = 35.26^0$, the second derivative of (5.27) is negative. Therefore, for the "Mercedes" probe with ideal (infinite) sensors, at these wires angle provides the largest uniqueness cone of 35.26^0 half-angle.

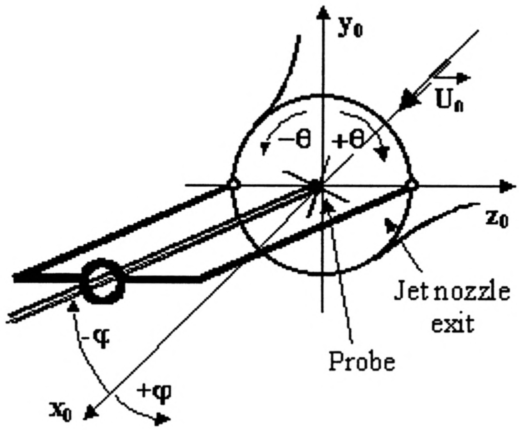


Fig. 11 A sketch of the coordinate system $Ox_0y_0z_0$, fixed to a test-rig, and a mechanism enabling angular probe calibration by φ and θ angle variation.

It follows, from the theoretical point of view, that "T" probe comparing to "*Mercedes*" probe consequently decreases the uniqueness cone for about $35.26^0 - 26.5^0 \sim 9^0$. However, "T" probes are less exposed to the influence of the fluid velocity gradients, in a boundary layer for example, because of the smaller vertical dimension (**Vukoslavčević and Petrović 1997**).

5.4 A proposal of a mechanism for testing the uniqueness ranges of 3-D probes

Presented analytical approach can be tested experimentally in a calibration mechanism shown in Fig. 11. Varying the angles φ and θ, different flow realizations of U, V and W can be induced, according to the formulas:

$$U = U_0 \cdot cos\varphi, \quad (5.30)$$

$$V = U_0 \cdot sin\varphi \cdot cos\theta, \tag{5.31}$$

$$W = U_0 \cdot sin\varphi \cdot sin\theta, \tag{5.32}$$

and effective cooling velocities measured. The uniqueness domain border is determined by varying the angle φ, for a given θ or the ratio W/V, until the critical value $\varphi = \varphi_{cr}$, defined by (5.8), is achieved.

6. FUNDAMENTALS OF THE NUMERICAL SIMULATION OF "T" AND "+" HOT-WIRE PROBE RESPONSES

To check the angular response of hot-wire probe, fluid velocity vectors of different orientations have been simulated. The unity magnitude of fluid velocity vector was assumed, $U_0 = 1$, because it is not of interest in these theoretical analysis. The angles θ and φ, which defines the ratios of simulated components U, V and W were varied in the ranges $0^0 - 360^0$ and $0^0 - 60^0$, respectively. On the base of the components U, V, W calculated according to (5.30-5.32), the effective cooling velocities U_{e1}, U_{e2}, U_{e4} that represents the response of the simulated ideal "T" probe (Fig. 9a) were evaluated according to expressions (4.1), (4.2) and (4.4).

After simple rearranging, (4.2) and (4.4) give, respectively:

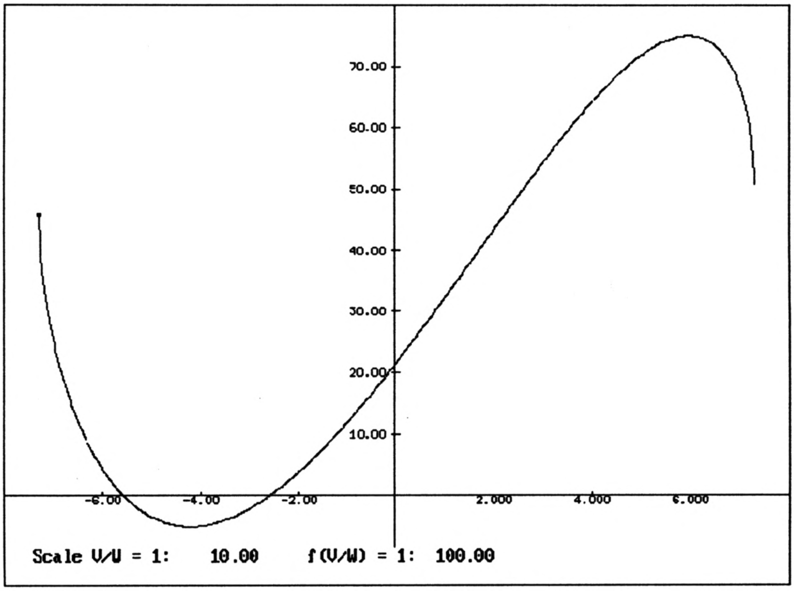
$$\sqrt{U_{e2}^2 - V^2} = U \cdot cos\alpha - W \cdot sin\alpha, \tag{6.1}$$

$$\sqrt{U_{e4}^2 - V^2} = U \cdot cos\alpha + W \cdot sin\alpha. \tag{6.2}$$

These expressions enable formulation of the longitudinal U and lateral W fluid velocity component as functions of the transversal V component and effective cooling velocities for the sensors no. 2 and 4:

$$U = \frac{1}{2 \cdot cos\alpha} \cdot \left(\sqrt{U_{e2}^2 - V^2} + \sqrt{U_{e4}^2 - V^2}\right), \tag{6.3}$$

$$W = \frac{1}{2 \cdot sin\alpha} \cdot \left(\sqrt{U_{e2}^2 - V^2} - \sqrt{U_{e2}^2 - V^2}\right). \tag{6.4}$$


70.00
60.00
50.00
40.00
30.00
20.00
10.00
-6.00
-4.00
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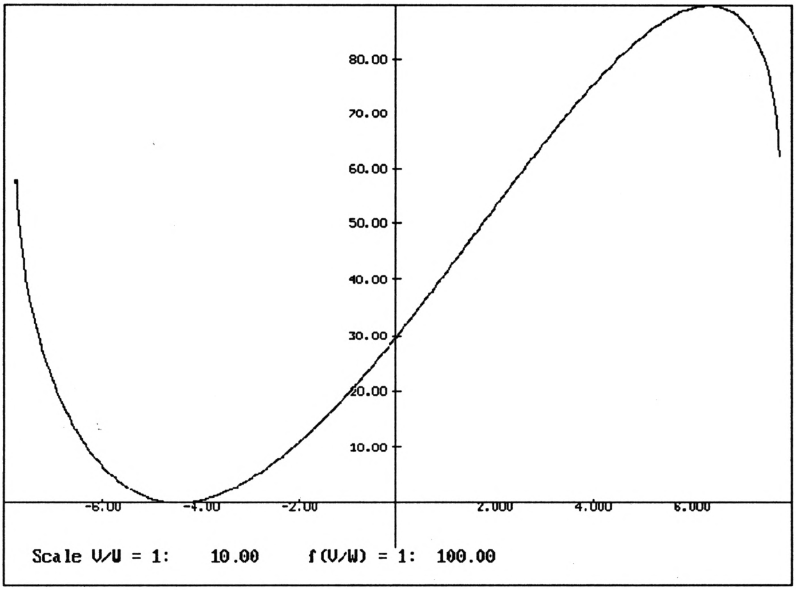
2.000
4.000
6.000
Scale V/U = 1: 10.00 f(V/W) = 1: 100.00

(a)


80.00
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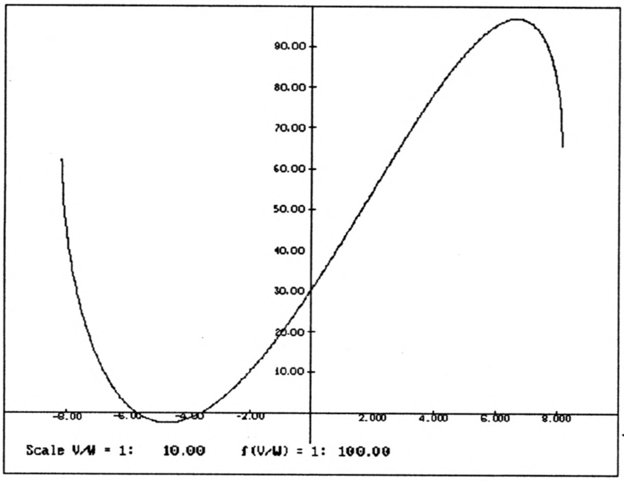
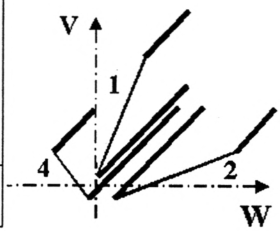
6.000
Scale V/U = 1: 10.00 f(V/W) = 1: 100.00

(b)





(c) (d)

Fig. 12: Functions $F = F(V)$, for the ideal "T" probe possessing infinite wires: (a) $\varphi = -15.0^0$, (b) $\varphi = -26.5^0$ and (c) $\varphi = -35.0^0$ and (d) the orientation of a simulated probe of "T" configuration.

After introducing the (6.3) and (6.4) in (4.1), the following function $F(V)$ arises:

$$F(V) = U_{e1}^2 - \left[\frac{1}{2}\left(\sqrt{U_{e2}^2 - V^2} + \sqrt{U_{e4}^2 - V^2}\right) - V sin\alpha\right]^2 -$$

$$- \frac{1}{4 \cdot sin^2\alpha}\left(\sqrt{U_{e4}^2 - V^2} + \sqrt{U_{e2}^2 - V^2}\right)^2 . \qquad (6.5)$$

This "characteristic" function can be used for the analysis of the uniqueness domain of hot-wire probe. In the regular situations, a set of 2 solutions usually exists. However, in the critical situation, just at the border of uniqueness domain, a single root exists only. This property of $F(V)$ is used in order to find the angular domain of the probe applicability. Of course, after evaluating the adequate root of (6.5), i.e. the value of the transversal V fluid velocity component, the U and W component can be evaluated from (6.3) and (6.4).

Fig. 12 presents the characteristic curves $F = F(V)$ for different ratios V/U and $W = 0$. It clearly shows that the curve $F = F(V)$ moves up and down, depending on the angle φ of the induced fluid velocity vector toward the probe axis. At small angles of the fluid velocity, like $\varphi = -15^0$ that belongs to the uniqueness domain, the curve $F(V)$ has 2 real roots, false $V = -0.558$ and true one $V = -0.259$ of the smaller absolute value. Increasing φ moves the $F(V)$ up. Thus, at the critical jaw angle $\varphi = -26.5^0$, just at the border of the uniqueness domain, $F(V)$ touches the abscissa, giving a single root $V = -0.447$.

After this point, further increasing the φ moves the curve $F(V)$ down and two roots exist again. For example, at $\varphi = -35^0$, we have one true root $V = -0.574$ and a false $V = -0.355$. Unfortunately, out from the uniqueness domain, the root of the larger absolute value is true. Therefore, out from the uniqueness cone limited by half-angle $\varepsilon = 26.5^0$, the signal interpretation procedure is not capable to make clear distinction between the true and false solutions of hot-wire response equations of "T" probe. It means that each specific hot-wire probe has to be applied only within its uniqueness domain.

On the base of this property of $F(V)$ function, i.e. searching for the values of φ when the $F(V)$ touches the abscissa, we have tested the simulated response of a "T" probe and found the uniqueness range borders. Following **Vukoslavčević and Wallace 1996**, the "+" probe was also analyzed, treating it like two joint "T" configurations possessing the wires no. 1, 2, 4 or 2, 3, 4 (see Fig. 9a).

7. BASIC RESULTS OF THE NUMERICAL SIMULATION OF "T" AND "+" HOT-WIRE PROBE RESPONSES

The uniqueness domains and cones of the "T" and "⊥" probes with infinite ideal wires oriented at 45^0 toward the probe axis are presented in Fig. 13. The first of them, "⊥ 45", has the upper wire no. 1 in the vertical plane, while the other, "T45" has the lower wire no. 3. These are identical probes, but symmetrically oriented. Therefore, their uniqueness domains are also mutually symmetrical according to abscissa. However, each of these two domains is asymmetrical by itself, according to abscissa, because of the geometrical asymmetry of "T" configuration. Thus, on the side of existing hot-wire, the uniqueness domain reaches the larger jaw angle of $\varphi = 38.8^0$, while on the opposite

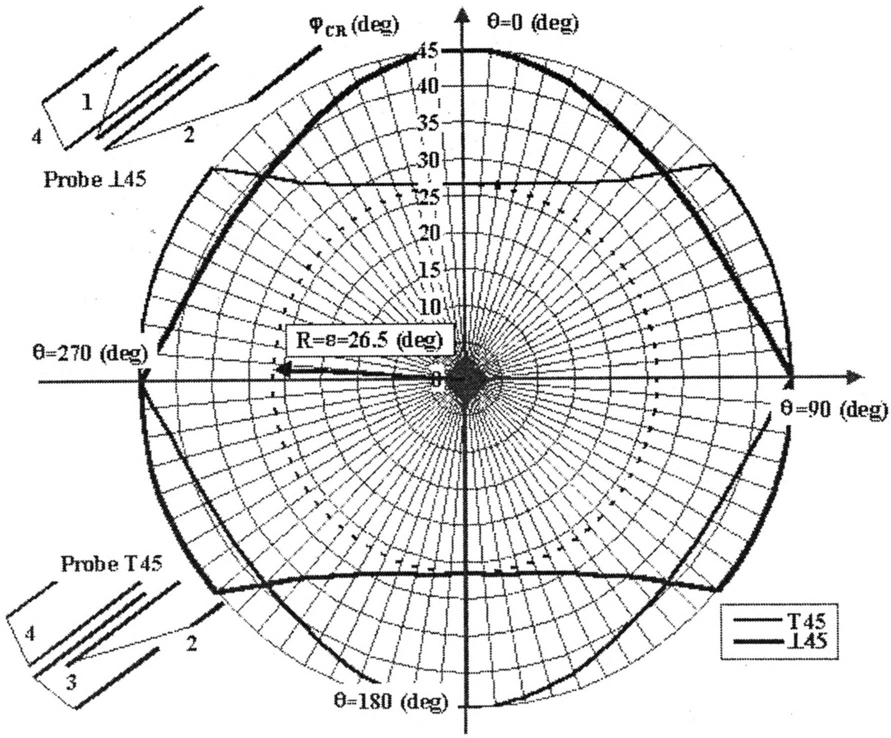


Fig. 13: The uniqueness domains of two symmetrically oriented triple-wire "T" configured ideal probes.

side this angle is only 26.5^0. For illustration, it can be mentioned that the larger value arises between $\theta = \pm 55^0, \pm 60^0$ and smaller at $\theta = 180^0$, for $\perp$ 45 probe (Fig. 13). For the T45 probe, situation is symmetrical according to abscissa. Obviously, the smaller φ value defines the half-angle of uniqueness cone of these probes, i.e. $\varepsilon = 26.5^0$.

These results are in agreement with the finding of **Vukoslavčević and Wallace 1983**. They experimentally confirmed the asymmetry of the uniqueness domain in total, as we did here in much more details by numerical simulation. The basic idea of **Vukoslavčević and Wallace 1996** was to design a symmetrical probe by introducing the 4^{-th} sensor to each of 3 arrays of the 9-wire probe. Consequently, the 12-sensor probe arose.

The new symmetrical geometry enabled them to develop the procedure that chooses the better one, between the 2 sensors lying in a vertical plane, for each instantaneous orientation of the non-stationary fluid velocity vector. Consequently, the 4-wire array was treated as 2 triple-sensor probes: "T45" and "⊥ 45" resolving the problem related to their geometrical asymmetry that causes the asymmetry of their uniqueness ranges.

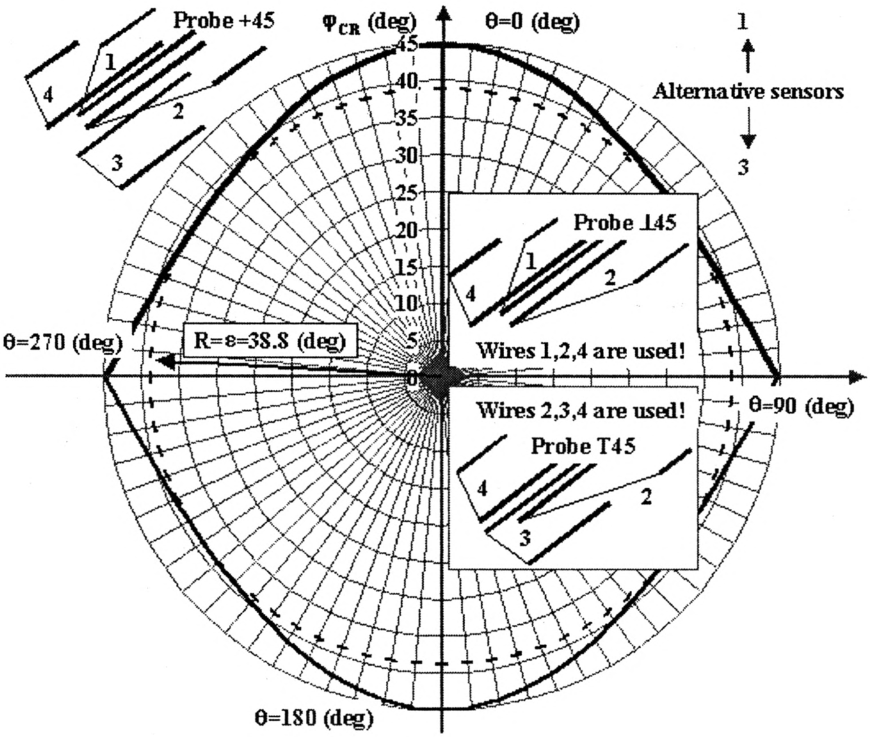


Fig. 14: The uniqueness cone of the "+" configured infinite sensors, altered in the vertical plane only, following the basic procedure for the twelve wire probe.

Presented results of our analysis show that this idea was correct. In the ideal case related to infinite sensors, illustrated in Fig. 14, the introduction of the 4^{-th} wire and involving the new interpretation procedure, enlarges the uniqueness cone from 26.5^0 in the case of "T" configuration with 3 wires, to 38.8^0 in the case of quadruple "+" configuration (Fig. 6).

As can be seen in figs. 13 and 14, the angular boundaries of quadruple configurations are symmetrical and, therefore, significantly over the boundaries of "T" geometry on the side of missing sensor. Consequently, this advanced property enlarged the uniqueness cone of 4-wire configurations. As it was expected, the angle 38.8^0 is also over the analytically found value of 35.26^0 for the half-angle of the uniqueness cone of the orthogonal triple-wire ("*Mercedes*") probe drawn in figs. 5c,d. To increase the accuracy of turbulent velocity field measurement, **Petrović 1996** have optimized the procedure for signal interpretation of quadruple hot-wire probes.

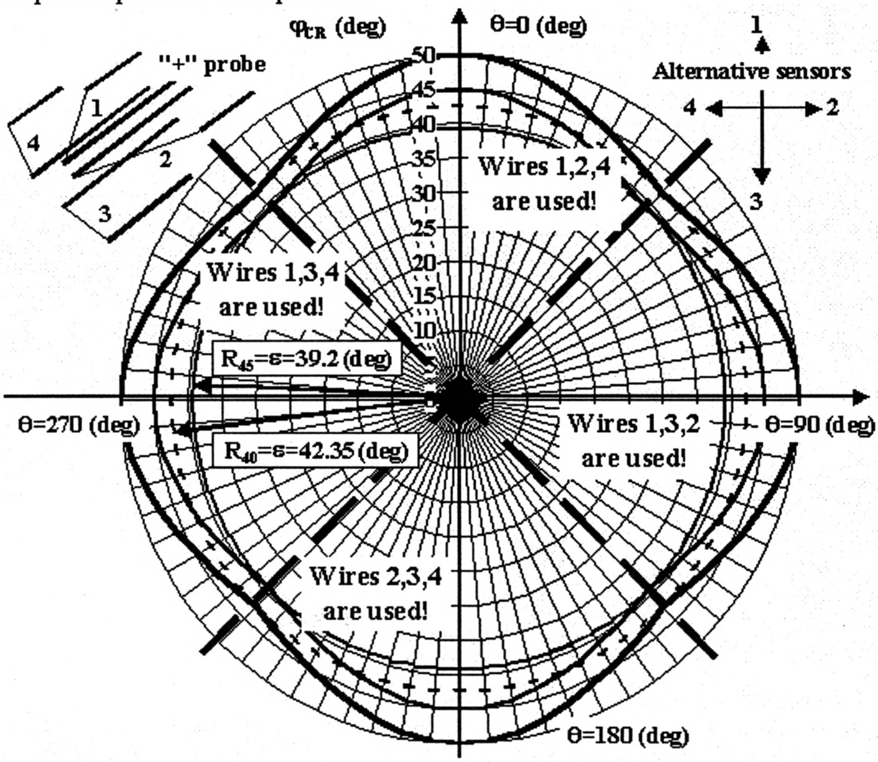


Fig. 15: The uniqueness cone of the ideal "+" configurations possessing wires mounted at geometrical angles of 45^0 and 40^0, altered in both the vertical and horizontal plane.

Among other advanced properties, his algorithm selects not only the better one between the 2 sensors placed in a vertical plane, as **Vukoslavčević and Wallace 1996** did, but chooses the 3 best sensors from the 4 available, including both the vertical and horizontal wires. Using his approach, 4 differently oriented "T" probes can be combined, giving uniqueness cone of 39.2^0 half-angle, for the probe possessing wires of 45^0 geometrical angle. The uniqueness domain of such probe is drawn by thin solid line in Fig. 15. The resulting half-angle of the uniqueness cone is enlarged for only $0.4^0(39.2^0 - 38.8^0)$, comparing to the basic procedure. Having in mind the expected negative influences of the end-wire cooling by cold prongs as well as of the aerodynamic blockage effect of prongs, this profit may be significantly decreased in the case of a real quadruple probe.

With intention to enlarge the uniqueness cone, some quadruple probe designers, like **Samet and Einav 1987** for example, have mounted the sensors at geometrical angle of 40^0. Consequently, the sensitivity of such configurations was insignificantly decreased in comparison to the sensitivity that can be achieved at 45^0. Following this idea, we have simulated the response of the ideal "T" and "+" probes (possessing infinite hot-wires) mounted at different geometrical angles. As it can be seen from the Fig. 15, the uniqueness domain of "+" probe possessing wires mounted at 40^0 is enlarged in comparison to configuration defined by wire angles of 45^0. The difference between their uniqueness cones is evident: 42.35^0 toward 39.2^0 half-angle, if approach of **Petrović 1996** is used for the signal interpretation. However, if the basic interpretation procedure is used, these angles are 41.8^0 and 38.8^0, respectively. Thus, decreasing the wire angle for 5^0 enlarges the uniqueness cone for about 3^0 in the ideal case, i.e. for infinite sensors.

This property is an important advantage of the quadruple probes in comparison to triple configurations, which angular domain of applicability can not be enlarged by increasing the wire angle over 35.26^0 for "*Mercedes*" configuration and over 45^0 for "T" probes. However, these conclusions are based on the idealized problem, what means that the uniqueness cone enlarging will be smaller in the practice, i.e. for the real probes. Still, the presented results are useful, because they provides the clear trends for further possible improving the design of quadruple probes.

8. CONCLUSIONS

The proposed approach is very convenient to define the asymmetry of the uniqueness domain, as a function of the ratio W/V, of different types of multi-sensor probes. The minimum value of the angle φ_{cr}

determined for each specific polar angle θ in the whole range $0^0 - 360^0$, defines the uniqueness cone half-angle ε, for a given type of the probe. Results reported by other authors, has been confirmed in the case of triple wire probes. A detailed analysis of the 4-sensor probe, based on the choice of 3 optimal out of 4 available sensors is presented. Altering only 2 vertical or horizontal sensor, mounted at geometrical angle of 45^0, the angular range is increased for $38.8^0 - 26.5^0 = 12.3^0$ comparing to the "T" probe, and $38.8^0 - 35.26^0 = 3.5^0$ according to the triple orthogonal probe. Using the optimized approach in selecting the best sensor combination, the uniqueness range is additionally increased for only 0.4^0, again for the wire geometrical angle of 45^0. The proposed approach is also convenient to analyze the influence of the wire inclination angle on uniqueness range. Reducing this angle from 45^0 to 40^0 increases the minimal angle of uniqueness cone for $41.8^0 - 38.8^0 = 3^0$ only in the case of 4-sensor probe, confirming the approach of **Samet and Einav 1987**, who have designed their probe using the wire angle of 40^0, although without any explanation. In the case of triple sensor probe the effect of reducing the inclination angle is opposite. Presented results have to be experimentally verified and corrected to real values that can be achieved in the practice. In the real situation the range will be affected by many additional parameters, at first place by the probe size and velocity magnitude. However, we believe that presented results still provide a clear guideline in the probe design at current borders of existing technology.

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