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ONE IMPROVED METHOD OFFERS WAY TO OPTIMIZE LOOPED PIPELINE NETWORK ANALYSIS

Abstract

This paper describes the iterative Hardy Cross method for determining the optimal hydraulic solution to the looped gas networks of conduits. This method is given in two forms: original Hardy Cross method also known as successive solution methods and improved Hardy Cross method also known as simultaneous solution method.

Keywords: Hardy Cross method, Looped pipeline network

JEDAN POBOLJŠANI METOD ZA OPTIMALNIJU ANALIZU PRSTENASTIH CEVOVODNIH MREŽA

Sažetak

U radu se opisuje iterativni Hardi Kros metod koji služi za nalaženje optimalnog hidrauličkog rešenja jedne gasovodne cevovodne mreže prstenastog tipa. Metod se daje u dve varijante, i to kao originalni Hardi Kros metod koji je takođe poznat kao metod uzastopnih rešenja i poboljšani Hardi Kros metod koji je takođe poznat kao metod istovremenih rešenja.

Ključne reči: Hardi Kros metod, Prstenasta cevovodna mreža

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1. INTRODUCTION

Since the value of the hydraulic resistance depends on flow rate, problem of flow distribution per pipes in a gas or water distributive looped pipeline networks has to be solved using some kind of iterative procedures. Such methods can be divided into two groups (1) Methods based on solution of the loop equations, and (2) Methods based on solution of the node equations. Most of the methods used commonly in engineering practice belong to the group based on solution of the loop equations. The Hardy Cross method from 1936 was the first useful method for the calculation of flow distribution or for calculation of optimal pipes diameters in the looped pipeline network [1]. Few iterative methods for these kind of calculations for water or gas looped pipeline networks, such as, the Hardy Cross [1], the modified Hardy Cross [2], the node-loop method [3, 4], the node method [5], method of V. G. Lobačev [6] and the method of M. M. Andrijašev [7], are available. Methods also can be used for ventilation networks. Methods developed by Russian authors [6,7] are similar with original Hardy Cross method [1]. Contemporary with Hardy Cross, soviet author V. G. Lobachev [6] was being developed very similar method compared with to original Hardy Cross method [1]. M. M. Andrijašev method [7] was very often being used in Russia during the soviet era. According to this method, contour and loop are not synonyms (contours for calculations has to be chosen to include few loops and only by exception one). The modified Hardy Cross method proposed Epp and Fowler [2] which considers entire system simultaneously is also sort of loop methods. The node-loop method proposed by Wood and Charles [3] and later improved by Wood and Rayes [4] is combination of the loop and node oriented methods, but despite of its name is essentially belong to the group of loop methods. Only the node method proposed by Shamir and Howard [5] is real representative of node oriented method. The node method uses idea of Hardy Cross but to solve node equations instead the loop ones.

Some notes on both versions of Hardy Cross method can be found in the paper of Brkić [8].

The analysis of looped pipeline systems by formal algebraic procedures is very difficult if the systems are very complicated. Electrical models had been used in studying this problem in the time before advanced computer became available as background to support demandable numerical procedures [9, 10]. Hydraulic networks and electric circuits with diodes instead of resistors are comparable.

2. LOOPED PIPELINE NETWORK

For the loop oriented methods first Kirchhoff's law must be satisfied for all nodes in all iterations (1). Second Kirchhoff's law for each loop must be satisfied with acceptable tolerance at the end of the calculation (2). In this group can be sorted here presented Hardy Cross method in both versions.

In a loop oriented method, first initial flow pattern must be chosen to satisfied first Kirchhoff's law (1). Endless number of flow combinations can satisfy this condition (1) [11]. Only one set of flows satisfy both, first and second Kirchhoff's law. This set of flows are the solution to this problem. Example network with three loops are shown in the figure 1.

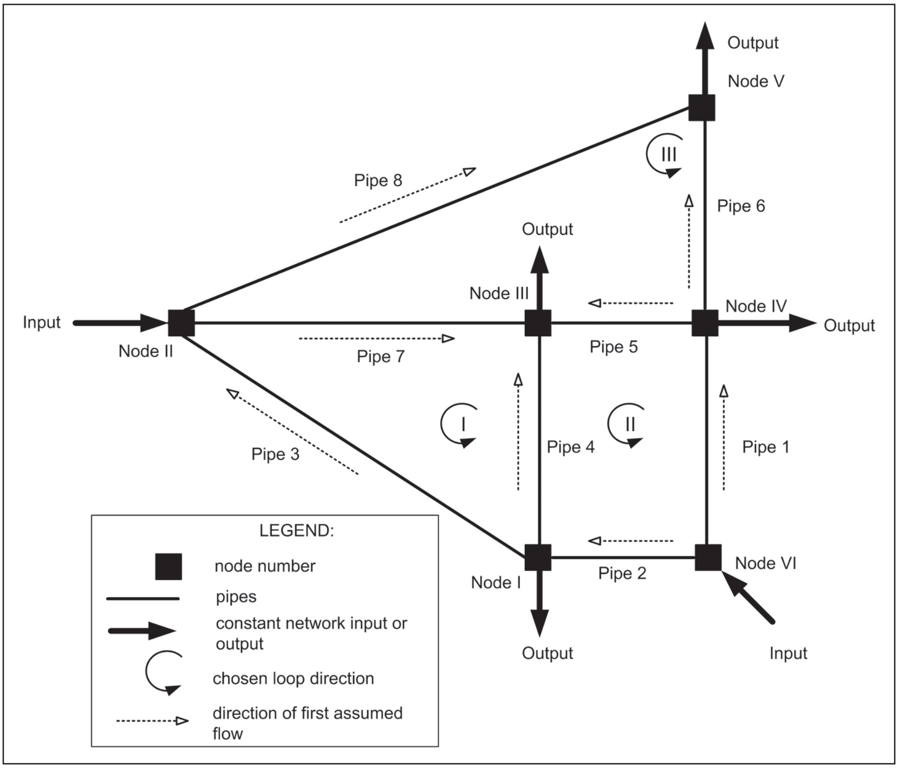

Figure 1. Example pipeline network with three loops

Nodes are sometimes also referred to as junctions, points or vertices while loops are sometimes referred also to as contour or path.

First Kirchhoff's law for the initial flow pattern shown in the figure 1 can be written using set of equations (1):

$$\begin{aligned}
&node_{I} \sim -Q_{I} + Q_2 - Q_3 - Q_4 = 0\\
&node_{II} \sim Q_{II} + Q_3 - Q_7 - Q_8 = 0\\
&node_{III} \sim -Q_{III} + Q_4 + Q_5 + Q_7 = 0\\
&node_{IV} \sim -Q_{IV} + Q_1 - Q_5 - Q_6 = 0\\
&node_{V} \sim -Q_{V} + Q_6 + Q_8 = 0\\
&node_{VI} \sim Q_{VI} - Q_1 - Q_2 = 0
\end{aligned} \tag{1}$$

Second Kirchhoff's law for the initial flow pattern shown in the figure 1 can be written using set of equations (2):

$$\begin{aligned}
&loop_{I} \sim -f_3 + f_4 - f_7 = F_{I}\\
&loop_{I} \sim f_1 - f_2 - f_4 + f_5 = F_{\mathbf{I}}\\
&loop_{III} \sim -f_5 + f_6 + f_7 - f_8 = F_{III}
\end{aligned} \tag{2}$$

Second Kirchhoff's law must be fulfilled for all loops at the end of calculation with demanded accuracy, i. e. $F_I \to 0$, $F_{II} \to 0$ and $F_{III} \to 0$.

For each pipe in the network f can be written function of pressure. For gas-lines this function represents pseudo-pressure drop (3) [12]:

$$f = p_1^2 - p_2^2 = \frac{4810 \cdot Q_n^{1.8} \cdot L \cdot \rho_r}{D_n^{4.8}} \tag{3}$$

Where p is pressure (Pa), Q is flow at normal pressure (m^3/s), D is inner pipe diameter (m), L is pipe length (m) and ρ_r is relative gas density (dimensionless).

For water-pipeline or other pipelines for transport of liquids is this function is pressure drop (4):

$$f = \Delta p = p_1 - p_2 = \lambda \cdot \frac{L}{D_n^5} \cdot \frac{8 \cdot Q^2}{\pi^2} \cdot \rho \tag{4}$$

Where p is pressure (Pa), λ is hydraulic resistance (dimensionless), Q is flow (m^3/s), D is inner pipe diameter (m), L is pipe length (m) and ρ is liquid density (kg/m^3). How to calculate hydraulic resistances can be seen e. g. in the paper of Yıldırım [13].

3. ORIGINAL VERSION OF THE HARDY CROSS METHOD

Hardy Cross, American engineer, developed method later named after him in 1936 [1]. According to this method correction of flow for each pipe in a particulate loop can be calculated after (5):

$$\Delta_1 = -\frac{F}{F'} \tag{5}$$

Where F is from (2), and F' is the first derivative of F (Q) where flow (Q) is treated as variable.

In presented example loop I begins and ends in node II via pipes 3, 4, and 7. For the loop I for the network from figure 1 this derivative is (6) for gas network and (7) for water network:

$$F_I' = \frac{\partial F_I(Q)}{\partial Q_I} = \frac{\partial\left(-f_3(Q_3)+f_4(Q_4)-f_7(Q_7)\right)_I}{\partial Q_I} = 1.82\cdot 4810\cdot\rho_r\cdot\left(-\frac{Q_3^{0.82}\cdot L_3}{D_3^{4.82}}+\frac{Q_4^{0.82}\cdot L_4}{D_4^{4.82}}-\frac{Q_7^{0.82}\cdot L_7}{D_7^{4.82}}\right)_I \tag{6}$$

$$F_I' = \frac{\partial f(Q)}{\partial Q_I} = \frac{\partial\left(-f(Q_3)+f(Q_4)-f(Q_7)\right)_I}{\partial Q_I} = \frac{16\cdot\rho}{\pi^2}\cdot\left(-\frac{\lambda_3\cdot L_3\cdot Q_3}{D_3^5}+\frac{\lambda_4\cdot L_4\cdot Q_4}{D_4^5}-\frac{\lambda_7\cdot L_7\cdot Q_7}{D_7^5}\right)_I \tag{7}$$

In the original Hardy Cross method [1], each loop correction is determined independently for each particular loop. Hence, the Hardy Cross method is also known as the single contour adjustment method.

For the first loop correction for gas network is (8):

$$(\Delta_I)_I = -\frac{F_I}{F_I'} = -\frac{4810\cdot\rho_r\cdot\left(-\frac{Q_3^{1.82}\cdot L_3}{D_3^{4.82}}+\frac{Q_4^{1.82}\cdot L_4}{D_4^{4.82}}-\frac{Q_7^{1.82}\cdot L_7}{D_7^{4.82}}\right)_I}{1.82\cdot 4810\cdot\rho_r\cdot\left(-\frac{Q_3^{0.82}\cdot L_3}{D_3^{4.82}}+\frac{Q_4^{0.82}\cdot L_4}{D_4^{4.82}}-\frac{Q_7^{0.82}\cdot L_7}{D_7^{4.82}}\right)_I} \tag{8}$$

In matrix form, original Hardy Cross method for the example network for gas distribution from figure 1 can be noted as (9):

$$\begin{bmatrix} \frac{\partial F_I(-Q_3,Q_4,-Q_7)}{\partial Q_I} & 0 & 0 \\ 0 & \frac{\partial F_{II}(Q_1,-Q_2,-Q_4,Q_5)}{\partial Q_{II}} & 0 \\ 0 & 0 & \frac{\partial F_{III}(-Q_5,Q_6,Q_7,-Q_8)}{\partial Q_{III}} \end{bmatrix} x \begin{bmatrix} (\Delta_I)_1 \\ (\Delta_{II})_1 \\ (\Delta_{III})_1 \end{bmatrix} = \begin{bmatrix} F_I \\ F_{II} \\ F_{III} \end{bmatrix} \quad (9)$$

Note, inner pipes in a network system are mutual to two loops and hence two different corrections calculated for two separate loops must be algebraically added in all iterations to flow rate assumed or calculated in previous iteration for these pipes. Goal is to preserve first Kirchhoff's law in all iteration for all nodes and finally to satisfy second Kirchhoff's law with acceptable tolerance.

4. IMPROVED VERSION OF THE HARDY CROSS METHOD

The improved or somewhere called the modified Hardy Cross method is also known as the simultaneous contour adjustment method [2]. As seen in figure 1, several loops have mutual pipes, so corrections to these loops will cause energy losses around more than one loop. In figure 1, pipe 4 belongs to two loops (loop I and II), pipe 7 to loop I and III, and finally pipe 5 to II and III. The modified Hardy Cross method is a sort of Newton–Raphson method used to solve unknown flow correction in one iteration taking into consideration whole system simultaneously [14]. Epp and Fowler [2] gave idea for this approach. To increase efficiency of the Hardy Cross method zeros from non-diagonal term in matrix equation (9) will be replaced to include influence of pipes mutual with adjacent loop (10):

$$\begin{bmatrix} \frac{\partial F_I(-Q_3,Q_4,-Q_7)}{\partial Q_I} & \frac{\partial F_I(-Q_4)}{\partial Q_{II}} & \frac{\partial F_I(Q_7)}{\partial Q_{III}} \\ \frac{\partial F_{II}(-Q_4)}{\partial Q_I} & \frac{\partial F_{II}(Q_1,-Q_2,-Q_4,Q_5)}{\partial Q_{II}} & \frac{\partial F_{II}(-Q_5)}{\partial Q_{III}} \\ \frac{\partial F_{III}(Q_7)}{\partial Q_I} & \frac{\partial F_{III}(-Q_5)}{\partial Q_{II}} & \frac{\partial F_{III}(-Q_5,Q_6,Q_7,-Q_8)}{\partial Q_{III}} \end{bmatrix} x \begin{bmatrix} (\Delta_I)_1 \\ (\Delta_{II})_1 \\ (\Delta_{III})_1 \end{bmatrix} = \begin{bmatrix} F_I \\ F_{II} \\ F_{III} \end{bmatrix} \quad (10)$$

First matrix in presented equation is symmetrical; for example (11):

$$\frac{\partial C_I(Q_7)}{\partial Q_{III}} = \frac{\partial C(Q_7)}{\partial Q_I} \quad (11)$$

This is because pipe 7 is mutual for two adjacent loops (loop I and loop III). Non-diagonal terms have always opposite sign than diagonal. Spatial networks common for e. g. ventilation systems in buildings or mines are exceptions [8, 15, 16].

5. ALGEBRAIC RULES FOR FLOW CORRECTIONS

Results of calculation using the original Hardy Cross or the improved Hardy Cross are not flows. Results are corrections of flows calculated for each loop. These corrections have to be added algebraically to flows from previous iteration for each pipe according to specific rules (12):

$$Q_{i+1} = Q_i \overset{\sim 1}{\mp} \Delta_1 \overset{\sim 2}{\mp} \Delta_2 \tag{12}$$

Where i is mark for iteration, Δ_1 is correction of flow from source loop and Δ_2 is correction from adjacent loop.

A pipe common to two loops receives two corrections. To understand better concept of corrections term upper and lower sign plus and minus will be introduced. The upper plus or minus sign indicates direction of flow in that conduit in these two pipes and is obtained from Q for previous iteration. The upper sign is the same as the sign in front of Q if the flow direction in each pipe coincides with the assumed flow direction in the particular pipe under consideration, and opposite if it does not. The lower sign is copied from the primary pipe for this correction (sign from the contour where this correction is first, sign preceding the first iteration from adjacent pipe for the conduit taken into consideration). The rules for sign of corrections are:

(1) the algebraic operation noted as $\overset{\sim 1}{\mp}$ in (12) for correction 1 should be the opposite of its sign; i. e. add when the sign is minus.

(2) the algebraic operation noted as $\overset{\sim 2}{\mp}$ in (12) for corrections 2 should be the opposite of their lower signs when their upper signs are the same as the sign in front of Q, and as indicated by their lower signs when their upper signs are opposite to the sign in front of Q.

For details of sing of corrections consult paper of Brkić [8] and chapter by Corfield et al. from Gas Engineers Handbook [17].

6. OPTIMIZED DESIGN OF LOOPED PIPELINE NETWORK

In the problem of optimization of pipe diameters, flow rates calculated previously or assumed are not any more treated as variable. In optimization problem pipes diameters are treated as variable and for gas network first derivative is now (13):

$$\frac{\partial\left(p_1^2 - p_2^2\right)}{\partial D_{in}} = \frac{\partial F}{\partial D_{in}} = \frac{\partial\left(\frac{4810 \cdot Q_{st}^{1.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}}\right)}{\partial D_{in}} = \frac{-4.82 \cdot 4810 \cdot Q_{st}^{1.82} \cdot L \cdot \rho_r}{D_{in}^{5.82}} \quad (13)$$

All previous calculations are still valid. Only corrections are not for flow but now for diameters. As the diameters are to be chosen among a finite set of available nominal values, this optimization problem is highly combinatorial.

7. CONCLUSION

The most famous method for solving of looped pipeline problems is the Hardy-Cross method, which was firstly devised for hand calculations, in 1936 [1]. This method today has only great historical and teaching value as alma mater of all today available and more efficient methods. The improved Hardy Cross method is very efficient but problem is with complicated algebraic rules which are used in calculating procedure [8, 17].

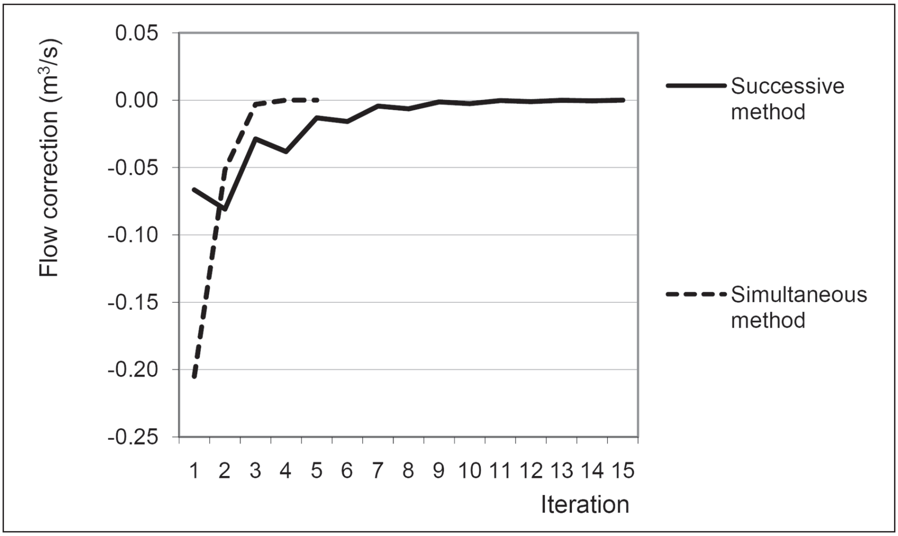

Figure 2. Flow correction in iterative procedure using original and improved Hardy Cross method (example from a real network)

This paper proves that the Hardy Cross method and diameter correction procedure can give good results when designing a gas-pipeline (or waterwork) network of composite structure. According to the price and velocity limits, the optimal design can be predicted. But all parameters, e. g. friction factor, relation for calculation of pressure drop in pipes, equation for calculation of gas flow, and similar must be chosen in a very careful way. In improved method performances of convergence is significantly incresed compared to the original version from 1936 (Figure 2).

Today, more efficient methods such as the node-loop exist [3, 4]. The node-loop method in which two topology matrices, i. e. the node and the loop matrix are united in coherent procedure for solution of looped pipeline problem directly with no correction involved. The node-loop method has even better convergence properties even in comparisons to the modified Hardy Cross method. Same network as in this paper are used to explain the node-loop method in the paper of Brkić [18].

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References

[1] Cross H (1936) Analysis of flow in networks of conduits or conductors. Engineering Experimental Station 286(34): 3–29
[2] Epp R, Fowler AG (1970) Efficient code for steady flows in networks. J Hydraul Div ASCE 96(1): 43–56
[3] Wood DJ, Charles COA (1972) Hydraulic network analysis using linear theory. J Hydraul Div ASCE 98(7): 1157–1170
[4] Wood DJ, Rayes AG (1981) Reliability of algorithms for pipe network analysis. J Hydraul Div ASCE 107(10): 1145–1161
[5] Shamir U, Howard CDD (1968) Water distribution systems analysis. J Hydraul Div ASCE 94: 219–234
[6] Lobačev VG (1934) New method for calculation of looped waterworks networks. Vod i San Tekhnika 2: 8–12
[7] Andrijašev MM (1932) Water systems calculation technique. 1 st ed. Moscow: Сов. зак–во
[8] Brkić D (2009) An Improvement of Hardy Cross method applied on looped spatial natural gas distribution networks. Appl Energ 86(7-8): 1290-1300

[9] Lilić BS (1956) Flow distribution in hydraulic networks. Dissertation, University of Belgrade (in Serbian) /also book from 1958/
[10] Mah RSH, Shacham M (1978) Pipeline network design and synthesis. Adv in Chem Eng 10: 125-209
[11] Gay B, Middleton P (1971) The solution of pipe network problems. Chem Eng Sci 26(1): 109-123
[12] Coelho PM, Pinho C (2007) Considerations about equations for steady state flow in natural gas pipelines. J Braz Soc Mech Sci Eng 29(3): 262–273
[13] Yıldırım G (2009) Computer-based analysis of explicit approximations to the implicit Colebrook–White equation in turbulent flow friction factor calculation. Adv Eng Softw 40(11): 1183–1190
[14] Mah RSH, Lin TD (1980) Comparison of modified Newton's methods. Comput Chem Eng 4(2): 75-78
[15] Aynsley RM (1997) A resistance approach to analysis of natural ventilation airflow networks. J Wind Eng Ind Aerod 67-68: 711-719
[16] Wang Y-J, Hartman HL (1967) Computer solution of three-dimensional mine ventilation networks with multiple fans and natural ventilation. Int J Rock Mech Min Sci 4(2): 129-154
[17] Corfield G, Hunt BE, Ott RJ, Binder GP, Vandaveer FE (1974) Distribution design for increased demand. In: Segeler CG (ed) Gas Engineers Handbook, Industrial Press, New York, pp 63–83
[18] Brkić D (2009) Gas distribution network topology problem, MICOM Ohrid, MASSEE International Congress on Mathematics, Organized by Mathematical Society of South-Eastern Europe and Union of Mathematicians of Macedonia, 16–20 September